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ONE-DIMENSIONAL PULSATION OF A TOROIDAL GASEOUS CAVITY IN A COMPRESSIBLE LIQUID

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Let us discuss within the framework of the acoustic approximation the problem of the pulsation of a toroidal cavity formed as a result of the explosion of a ring-shaped explosive charge on condition of the fulfillment of the inequality $a \gg R$, where a = const is the radius of the torus and R is the radius of the cavity. At the same time the cross section of the toroidal cavity practically preserves the shape of a true circle, as the experimental data show, during a single pulsation period when $a \approx 10^3 R_*$ and during a single half-period of pulsation when $a \approx 10^2 R_*$. (R_* is the radius of the charge). The problem of the pulsation of a gaseous torus in an incompressible liquid has been discussed in [1]; however, it does not offer the possibility of evaluating such an important parameter as the maximum radius of the expanding cavity, and consequently, the energy distribution among the detonation products and the shock wave in the case of an explosion with axial symmetry.

The solution of the indicated problem is fraught with many difficulties, in particular, the complexity of the solution of the wave equation. Therefore, it is necessary first of all to find a method of constructing an equation of one-dimensional pulsation which would permit simplifying the problem posed. Since an expression for the velocity potential can be found for a number of spatial potential problems of an ideal incompressible liquid in the case of specified assumptions, an attempt to use it for the transition to acoustic models is natural. The practicability of this method is shown below in the example of the construction of the equation of one-dimensional pulsation of bubbles.

§1. Let the velocity potential in the case of an incompressible liquid have the form $\varphi = \Phi(t)/f(r)$. Then its acoustic version can be represented as $\varphi = \Phi(t - r/c_0)/f(r)$. Since potential flow of a liquid $u = -\nabla\varphi$ is being discussed, where u is the velocity of a fluid particle, then

$$u = \Phi f_r/f^2 + \Phi'/c_0 f, \tag{1.1}$$

where the prime denotes a derivative with respect to $\zeta = t - r/c_0$. The Cauchy – Lagrange integral with the form of φ taken into account can be written as

$$\Phi' = f(\omega + u^2/2), \tag{1.2}$$

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where $\omega = \int dp/\rho$, p is the pressure in the liquid, and ρ is its density. From (1.1) and (1.2) it is possible to find an expression for Φ

$$\Phi = f^2[u - (\omega + u^2/2)/c_0]/f_r.$$

Let us take the derivative of this expression with respect to t, and we obtain

$$\Phi' = f^2[u_t - (\omega_t + uu_t)/c_0]/f_r. \tag{1.3}$$

On the basis of the continuity equations (acoustic version) and momentum conservation

$$u_r + \nu u/r = -(d\omega/dt)/c_0^2, \quad \partial\omega/\partial r = -du/dt,$$

where $\nu = 0, 1, 2$ are for the two-dimensional, cylindrical, or spherical cases, respectively, let us find expressions for the partial derivatives u_t and ω_t in (1.3). They are of the form

$$u_t = du/dt + \nu u^2/r + u(d\omega/dt)/c_0^2, \tag{1.4}$$
$$\omega_t = d\omega/dt + u(du/dt).$$

Substituting (1.4) into (1.3) and equating the expression obtained to Eq. (1.2), we finally obtain

$$f(1 - 2u/c_0)\,du/dt + \nu f u^2(1 - u/c_0 - rf_r/2\nu f)/r = \omega f_r + f(d\omega/dt)(1 - u/c_0 + u^2/c_0^2)/c_0. \tag{1.5}$$

In (1.5) it is possible to proceed to the cavity wall by setting $r = R$, $u = dR/dt$, and $\omega = [p(R) - p_\infty]/\rho_0$, where $p(R)$ is the pressure in the cavity, p_∞ is the pressure at infinity, and ρ_0 is the density of the undisturbed liquid. Thus we have for a two-dimensional cavity ($\nu = 0$, $f = 1$, $f_r = 0$)

$$[1 - 2(dR/dt)/c_0]d^2R/dt^2 = [1 - (dR/dt)/c_0 + (dR/dt)^2/c_0^2](d\omega/dt)/c_0;$$

for a spherical cavity ($\nu = 2$, $f = 4$, $f_r = 1$)

$$R[1 - 2(dR/dt)/c_0]\,d^2R/dt^2 + (3/2)(dR/dt)^2[1 - 4(dR/dt)/3c_0] =$$
$$= \omega + (R/c_0)(d\omega/dt)[1 - (dR/dt)/c_0 + (dR/dt)^2/c_0^2];$$

and for a cylindrical cavity ($\nu = 1$, and using the Rice–Ginell modification of the Kirkwood–Bethe approximation [2], we set $f = r^{1/2}$ and $f_r = r^{-1/2}/2$)

$$R[1 - 2(dR/dt)/c_0]\,d^2R/dt^2 + (3/4)(dR/dt)^2[1 - 4(dR/dt/3c_0] =$$
$$= \omega/2 + (R/c_0)(d\omega/dt)[1 - (dR/dt)/c_0 + (dR/dt)^2/c_0^2].$$

All three equations correspond exactly to the equations derived on the basis of the Kirkwood–Bethe approximation [2] for the acoustic case

$$(\partial/\partial t + c_0\partial/\partial r)G = 0,$$

where $G = r^{\nu/2}(\omega + u^2/2)$, which is also derived in the method expounded above.

The proposed method of finding the pulsation equation of a cavity is rather simple. However, it still cannot be used in two-dimensional problems because of the fact that in this case the continuity equation does not allow replacing partial derivatives of the velocity components by the total derivatives. Analysis of the method has shown that it allows a simplification which reduces to the following. Let us assume that it is possible to neglect in the continuity equation the term $(d\omega/dt)/c_0^2$, i.e., to assume that the relation among the velocity components is determined essentially by the limits of an ideal incompressible liquid. At the same time, if a solution of the Laplace equation for the velocity potential is found and the boundary conditions affiliated with the corresponding statement of the problem permit separation of variables, each velocity component is expressed in terms of a total derivative of the cavity radius with respect to t.

One can show that the assumption made has an insignificant effect on the form of (1.5): The terms u/c_0 and u^2/c_0^2 affiliated with $d\omega/dt$ on the right-hand side of this equation in parentheses disappear. But it is also possible to neglect them within the framework of acoustics, since the main losses to radiation are determined, in the case of the pulsation of a cavity in an incompressible liquid, by the term $(R/c_0)(d\omega/dt)$. We will use the results obtained to find the pulsation equation of a toroidal cavity in the acoustic approximation, and we will compare the pulsation parameters calculated from it with experimental data.

§2. Let there occur in a liquid a toroidal cavity formed as the result of the "instantaneous" explosion of a ring charge whose linear dimensions satisfy the inequality $a \gg R$. Then it is possible within the framework of the ring-source approximation to write the following expression for the velocity potential:

$$\varphi = -(a/2\pi)\int_0^{\pi}\Phi(t - f/c_0)\,d\alpha/f, \tag{2.1}$$

where $\Phi = dS/dt$; $S = \pi R^2$ is the cross-sectional area of the torus, $f = \sqrt{z^2 + r^2 + a^2 - 2ar\cos\alpha}$ in the cylindrical coordinate system (z, r, α), and α is figured from an arbitrarily selected direction in the plane of the torus. According to Sec. 1, it would be possible to write out in explicit form an expression for Φ in the case of an incompressible liquid. It is not clear in this connection how to write the argument of the function Φ upon a transition to the acoustic model. Therefore, let us preserve the expression for the velocity potential in the form (2.1), having stipulated that it is possible to remove the function Φ' from under the integral sign. This assumption is not essential to the construction of the pulsation equation of a cavity and can have an effect only in the evaluation of the fine structure of a shock wave in the zone next to the charge.

By analogy with what has been stated above it is possible to write

$$\Phi' = (2\pi/a)(\omega + V^2/2)\Big/\left(\int_0^{\pi} d\alpha/f\right), \tag{2.2}$$

$$\frac{a}{2\pi}\int_0^{\pi}\frac{\Phi(f_r + f_z)}{f^2}\,d\alpha = u + v - \frac{\omega + V^2/2}{c_0\left(\int_0^{\pi} d\alpha/f\right)}\int_0^{\pi}\frac{f_r + f_z}{f}\,d\alpha,$$

where u, v, and V are the components and the total velocity of a fluid particle. Let us take the partial derivative of the second equation in (2.2) with respect to t. Then, removing Φ' from under the integral sign, we obtain

$$\frac{a\Phi'}{2\pi}\int_0^{\alpha}\frac{f_r + f_z}{f^2}\,d\alpha = u_t + v_t - \frac{\omega_t + VV_t}{c_0\left(\int_0^{\pi} d\alpha/f\right)}\int_0^{\pi}\frac{f_r + f_z}{f}\,d\alpha.$$

It can be shown that $\omega_t = d\omega/dt + VdV/dt$. The partial derivatives u_t and v_t will be found from the solution for an incompressible liquid. At the same time, using the expression $\varphi = -(a/2\pi)\Phi\int_0^{\pi} d\alpha/f$, for the velocity potential, we finally obtain

$$\begin{gathered} u_t = du/dt - (a/2\pi)\Phi\left\{u\int_0^{\pi}(f_{rr}/f^2 - 2f_r^2/f^3)\,d\alpha + v\int_0^{\pi}(f_{rz}/f^2 - 2f_rf_z/f^3)\,d\alpha\right\}, \\ v_t = dv/dt - (a/2\pi)\Phi\left\{u\int_0^{\pi}(f_{rz}/f^2 - 2f_rf_z/f^3)\,d\alpha + v\int_0^{\pi}(f_{zz}/f^2 - 2f_z^2/f^3)\,d\alpha\right\}. \end{gathered} \tag{2.3}$$

Substituting the expressions for ω_t, u_t, and v_t obtained in (2.3) and expressing Φ' from the first Eq. (2.2), we obtain

$$\begin{aligned} &(1 - 2F_1u)du/dt + (1 - 2F_1v)dv/dt - (\pi/a)(F_0/F)(u^2 + v^2) + \\ &+ (a/2\pi)\Phi F_1F_2u^2 + (a/2\pi)\Phi F_1F_3v^2 + (a/\pi)\Phi F_1F_4uv - (a/2\pi)\Phi(F_2 + \\ &+ F_4)u - (a/2\pi)\Phi(F_3 + F_4)v = (2\pi/a)(F_0/F)\omega + F_1 d\omega/dt, \end{aligned} \tag{2.4}$$

where

$$F_0 = (a/2\pi)\int_0^{\pi} f^{-2}(f_r + f_z)\,d\alpha; \quad F = \int_0^{\pi} d\alpha/f;$$

$$F_1 = (c_0F)^{-1}\int_0^{\pi} f^{-1}(f_r + f_z)\,d\alpha; \quad F_2 = \int_0^{\pi} f^{-2}(f_{rr} - 2f_r^2/f)d\alpha;$$

$$F_3 = \int_0^{\pi} f^{-2}(f_{zz} - 2f_z^2/f)\,d\alpha; \quad F_4 = \int_0^{\pi} f^{-2}(f_{rz} - 2f_rf_z/f)\,d\alpha;$$

$$\Phi = 2\pi R\,(dR/dt).$$

In final form the functions F are written in the following way:

$$F_0 = \frac{a}{2\pi r\sqrt{z^2+(r+a)^2}}\left[\frac{2r^2-a^2-(z-r)^2}{z^2+(r-a)^2}E(k)+K(k)\right], \quad F = \frac{2K(k)}{\sqrt{z^2+(r+a)^2}},$$

$$F_1 = \frac{\pi\sqrt{z^2+(r+a)^2}}{4rc_0K(k)}\left[1+\frac{2r^2-a^2-(r-z)^2}{\sqrt{[z^2+(r+a)^2][z^2+(r-a)^2]}}\right], \quad k=\sqrt{\frac{4ar}{z^2+(r+a)^2}},$$

$$F_2 = -\frac{3K(k)}{2r^2\sqrt{z^2+(r+a)^2}}+\frac{3(z^2+a^2)-r^2}{r^2}\frac{E(k)}{[z^2+(r-a)^2]\sqrt{z^2+(r+a)^2}}-$$

$$-\frac{(z^2+a^2-r^2)^2\sqrt{z^2+(r+a)^2}}{2r^2[z^2+(r-a)^2]^2[z^2+(r+a)^2]^2}\{4(z^2+r^2+a^2)E(k)-[z^2+(r-a)^2]K(k)\},$$

$$F_3 = \frac{2E(k)}{[z^2+(r+a)^2]\sqrt{z^2+(r+a)^2}}-\frac{2z^2\sqrt{z^2+(r+a)^2}}{[z^2+(r-a)^2]^2[z^2+(r+a)^2]^2}\{4(z^2+$$

$$+r^2+a^2)E(k)-[z^2+(r-a)^2]K(k)\}, \quad F_4 =$$

$$= -\frac{3zE(k)}{r[z^2+(r-a)^2]\sqrt{z^2+(r+a)^2}}+\frac{z(z^2+a^2-r^2)\sqrt{z^2+(r+a)^2}}{r[z^2+(r-a)^2]^2[z^2+(r+a)^2]^2}\{4(z^2+r^2+$$

$$+a^2)E(k)-[z^2+(r-a)^2]K(k)\},$$

where K(k) and E(k) are the complete elliptic integrals of the first and second kinds, respectively, and k is their modulus. Proceeding to the wall of the torus, we obtain from (2.4) after a number of rearrangements in the expressions for the coefficients F the pulsation equation of a toroidal gaseous cavity in a compressible liquid

$$\{[1-2\pi(dR/dt)/c_0\ln(8a/R)]Rd^2R/dt^2+$$
$$+[1-\pi(dR/dt)/c_0\ln(8a/R)](dR/dt)^2\}\ln(8a/R)-(dR/dt)^2/2=$$
$$=\omega+\pi(R/c_0)(d\omega/dt). \tag{2.5}$$

On the basis of Eq. (2.5) a calculation has been carried out of the pulsation parameters of the toroidal cavity formed as the result of the explosion of an explosive ring (the explosive is hexogen, the density of the charge is $\rho_* = 1.55$ kg/cm^3, the detonation rate is D = 7.7 km/sec, and the charge diameter is d = 0.65 mm). Since the experimental investigations with which the calculated data are compared below were performed with charges having a copper shell and the finiteness of the detonation rate had practically no effect on the shape of the cavity, the initial parameters of the problem were determined from the decay condition of an arbitrary explosion in the case of "instantaneous" detonation, and the data of [3] were used for the isentropic index.

The results of the calculation are given in Table 1, where $a_0 = a/R_*$, $y_+^0 = R_+^0/R_*$, $y_-^0 = R_-^0/R_*$, R_+^0 and R_-^0 are the maximum and minimum radii of the cavity at the instants of the first expansion and the first collapse, respectively; t^0 is the time of expansion out to R_+^0 (one half of the first pulsation period); E is the fraction of the energy remaining in the detonation products after the first expansion of the cavity; and $a_0 = \infty$ corresponds to an infinite cylindrical charge.

It is possible on the basis of the calculational and experimental results presented in Table 1 to note the following:

a) as the radius of the ring charge increases, the values of the pulsation parameters y_+^0 and y_-^0, which characterize the energy balance of the explosion, asymptotically approach the corresponding values of the parameters of an explosion with cylindrical symmetry;

b) the calculated data agree satisfactorily with the experimental data for $a_0 > 3 \cdot 10^2$;

c) the ring geometry of the charge has a significant effect on the pulsation period of a cavity with detonation products: Even at a ring charge radius of a = 10 m (data under No. 6 in Table 1) the pulsation period of the

TABLE 1

No.	a_0	Calculation			Experiment		
		y_+^0	y_-^0	$t^0 \cdot R_*^{-1}$, sec/cm	y_+^0	$t^0 \cdot R_*^{-1}$, sec/cm	E, %
1	$1{,}54 \cdot 10^2$	125,83	4,02	0,21	103	0,21	11,8
2	$3{,}08 \cdot 10^2$	127,8	3,82	0,24	119,5	0,237	15,9
3	$4{,}60 \cdot 10^2$	128,8	3,72	0,256	123,6	0,256	17,0
4	$1{,}54 \cdot 10^3$	131,5	3,48	0,3	—	—	—
5	$3{,}08 \cdot 10^3$	132,8	3,37	0,323	—	—	—
6	$3{,}08 \cdot 10^4$	136,4	3,09	0,394	—	—	—
7	∞	140,9	3,15	0,2	135	0,2	22

torus exceeds by practically a factor of two its value for a cylindrical explosion; the experimental results confirm the tendency toward an increase of the pulsation period upon an increase in the radius of the ring; and

d) according to the experimental data, as the radius of the ring decreases (with fulfillment of the condition of maintaining the toroidal nature of the cavity), the fraction of energy necessary for the shock wave increases and amounts to practically 90% for a value $a_0 \approx 150$; as the radius of the ring increases, the energy balance approaches the data for an explosion with cylindrical symmetry.

The results presented for our investigations confirm the practicability of the method proposed in this paper and the pulsation equation (2.5) obtained on this basis for a toroidal cavity in a compressible liquid.

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SHOCK STRUCTURE IN A LIQUID CONTAINING GAS BUBBLES WITH NONSTEADY INTERPHASE HEAT TRANSFER

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In this article the one-velocity, two-pressure model of a two-phase mixture [1] is used in conjunction with the heat-conduction equation for the interior of bubbles in a bubble–liquid mixture to describe the structure of a shock wave in such a mixture.

Shock waves in a liquid containing gas bubbles have been investigated theoretically and experimentally [1-4]. The structure of a shock wave in such a medium has been studied with allowance for the compressibility of the host phase as well as two-velocity and two-temperature effects [5], and it has been shown in the same work that in the case of thermal nonequilibrium the role of two-velocity effects becomes inconsequential against the background of the much stronger thermal dissipation. In this connection the present discussion is framed in the one-velocity model for simplification [6]. The objective of the present study is to refine the results of [6] and to test the applicability of the fixed heat-transfer coefficient or Nusselt number determined from the approximation of a thin thermal boundary layer to the case of nonsteady heat transfer between a pulsating bubble and the host liquid.

§ 1. Fundamental Equations

We consider the motion of a liquid in which gas bubbles are suspended and for which the following basic assumptions are made [1]: 1) The distances over which the flow parameters experience any appreciable variation are much greater than the distances between bubbles, and the latter distances in turn are much greater than the bubbles themselves (i.e., the contents by volume α_2 of the gas phase are small, $\alpha_2 < 0.1$); 2) the mixture is monodisperse, i.e., in every elementary volume all the bubbles are spherical and have the same radius R; 3) viscosity and heat conduction are essential only in interphase processes and, in particular, during bubble pulsations.

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